Pedagogical Approaches to Research Concepts

Alexander Thomas

The goal should be, not to implant in the students' mind every fact that the teacher knows now; but rather to implant a way of thinking that enables the student, in the future, to learn in one year what the teacher learned in two years. Only in that way can we continue to advance from one generation to the next. - Edwin Jaynes
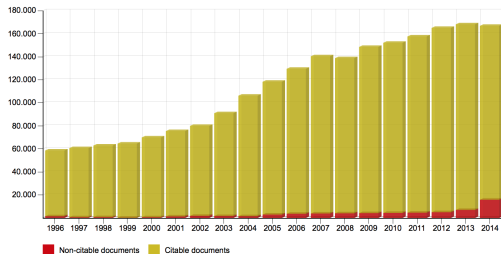
Annual BRCP Symposium 2021

Some facts

- Duration of PhD: today 3-4 years
  - Around 1850: 1-2 years
  - Median age of PhD in Germany in 2014: 30.4, in US: 32 [Wikipedia]
- Papers in pure maths in 2020: around 40,000 [arXiv]
  - Number of papers every day in a specific area: around 5-10
- Mean number of pages of a paper around 20-30 [own stat]

Pros	Cons
Research becomes too technical	New accessible areas
Growing distance to research boundary	Better understanding
Too many papers	New technologies

Possible ways out

- Make science more human-friendly
- Teach how to teach, more importance to teaching
- New technologies: video format, papers with annotations,...
- Material giving overviews, without technical details
  - valuate these activities (journals?)

Examples of sources

- Wikipedia
- Physics Travel Guide
- John Baez' blog "This weeks finds"
- other blogs: Timothy Gowers, Terence Tao, ...

Pharos Library Project

pharos-library.com

- Wiki to gather sources of pedagogical material (papers, videos, ...)
- Created this year by BRCP members
- Criteria: pleasant to read, overview, for researchers, visual (illustrations, diagrams) ,without technical details
- Everyone can modify and join!

Soergel bimodules

`Modules imitating the Iwahori-Hecke algebra`

Soergel bimodules were introduced in 1992 by Wolfgang Soergel to study representations of Lie groups[1].

Computing tensor products between Soergel bimodules is formally equivalent to computations in the Iwahori–Hecke algebra. In fancy language: Soergel bimodules are a categorification of Iwahori–Hecke algebras. This phenomenon explains positivity properties of the latter.

Graphical calculus between Soergel bimodules[2]
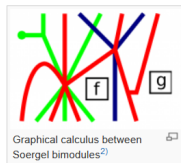
Here is a concrete example of this categorification: the Hecke algebra admits a basis (b_w) called the Kazhdan-Lusztig basis. For a simple basis element b_s we have $b_s^2 = (v + v^{-1})b_s$ where v is the formal parameter in the Hecke algebra. To each bw is associated a Soergel bimodule B_w. We have

$$B_s \otimes B_s \cong B_s(1) + B_s(-1)$$

where $B_s(1)$ is a shift of B_s. This shift corresponds to the multiplication by v.

Elias–Williamson have developped a graphical calculus for Soergel bimodules (see the figure for an example).

Edit

Material

A lovely introduction to Coxeter groups, the Hecke algebra and Soergel bimodules can be found here (Soergel bimodules in Section 4):

- Nicolas Libedinsky: *Gentle introduction to Soergel bimodules I: The basics*, https://nicolaslibedinsky.cl/gentle-introduction-to-soergel-bimodules-i-the-basics/, arXiv https://arxiv.org/abs/1702.00039

Edit

More ideas

The advantage in the landscape of modules is that one can speak about an *indecomposable module* (those which can not be written as direct sum of two others). These are precisely the Soergel bimodules with shift $B_w(k)$ and can be used as a natural basis in the space of bimodules.

Discussion

- How do you feel about this topic? Other viewpoints?

- Sources for accessible explanations for research topics?

- How to revalue pedagogic papers, blogs or videos?

- New technologies to make science more human-friendly?

- ...

References

- Wikipedia: Doctor of Philosophy
- PhD in 1840:
  https://physicstoday.scitation.org/do/10.1063/PT.5.9071/full/
- Number of articles: https://arxiv.org/year/math/20